Computational Challenge in the Coupled Filtering Method for Ultrasound Image-based Tissue Deformation Analysis

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Overview

- Background of tissue deformation analysis
- Feature-motion decorrelation problem in ultrasound speckle tracking
- Coupled filtering method and its merit
- Computational challenge
Background

Manual Palpation and Ultrasound Elasticity Imaging
Principle of Elastography

1. Take image $I_1$ before tissue deformation
2. Take another image $I_2$ after deformation
3. Calculate tissue displacement using $I_1$ and $I_2$
4. Derive strain values based on displacement

(From M. Insana., Ultrasonic Imaging Laboratory at UIUC)
Issues in Elastography

- Basic requirement: accurate estimation of tissue displacement
- Problem: feature-motion decorrelation
- Consequence: motion tracking results CANNOT reveal underlying tissue motion faithfully.
Echocardiography
Issues in Cardiac Motion Analysis

- Popular approach: speckle tracking
- Fundamental assumption: speckles are motion-invariant
- Problem: feature-motion decorrelation again!
Feature-Motion Decorrelation

- Why do we have this phenomenon?
- When will it happen?
- Can we compensate for the decorrelation?

Answers lie in the understanding of ultrasound imaging process.
Principle of Ultrasound Imaging
A Linear Convolution Model of Ultrasound Imaging Process

- Point Spread Function
  \[ H(\vec{X}) = e^{-\frac{1}{2} \vec{X}^T \Gamma \vec{X}} \cdot \cos(2\pi \vec{X}^T \vec{U}_0). \]

- Scatterer Model
  \[ T_n(\vec{X}; \vec{X}_n) = a_n \delta(\vec{X} - \vec{X}_n) \]

- RF Signal
  \[
  I(\vec{X}; \vec{X}_n) = \sum_{n=1}^{N} T_n(\vec{X}) \ast H(\vec{X}; \vec{X}_n)
  = \sum_{n=1}^{N} a_n e^{-\frac{1}{2} (\vec{X} - \vec{X}_n)^T \Gamma (\vec{X} - \vec{X}_n)} \cdot \cos(2\pi (\vec{X} - \vec{X}_n)^T \vec{U}_0).
  \]

- B-Mode Signal
  \[ I_B(X; \vec{X}_n) = |I_A(\vec{X}; \vec{X}_n)|. \]

with
\[
I_A(\vec{X}; \vec{X}_n) = \sum_{n=1}^{N} a_n e^{-\frac{1}{2} (\vec{X} - \vec{X}_n)^T \Gamma (\vec{X} - \vec{X}_n)} \cdot e^{j2\pi (\vec{X} - \vec{X}_n)^T \vec{U}_0}.
\]

(Meunier, Physics in Medicine and Biology, 43:1241-1254, 1998)
Intensity before and after Tissue Motion

- **Tissue Motion Model:**
  \[ \tilde{X}_n' = M \tilde{X}_n = \begin{pmatrix} R_{3 \times 3} & \bar{D}_{3 \times 1} \\ \bar{O}_{1 \times 3} & 1 \end{pmatrix} \tilde{X}_n \]

- **Tissue motion:**
  \[ I_A(\tilde{X}; M \tilde{X}_n) = \sum_{n=1}^{N} a_n e^{-\frac{1}{2} (M^{-1} \tilde{X} - \tilde{X}_n)^T M^T \Gamma M (M^{-1} \tilde{X} - \tilde{X}_n)} \cdot e^{j2\pi (M^{-1} \tilde{X} - \tilde{X}_n)^T M^T \bar{U}_0} \]

- **Geometric transform**
  \[ I_A(M^{-1} \tilde{X}; \tilde{X}_n) = \sum_{n=1}^{N} a_n e^{-\frac{1}{2} (M^{-1} \tilde{X} - \tilde{X}_n)^T \Gamma (M^{-1} \tilde{X} - \tilde{X}_n)} \cdot e^{j2\pi (M^{-1} \tilde{X} - \tilde{X}_n)^T \bar{U}_0} \]

- **No Feature-Motion Decorrelation Means:**
  \[ I_A(\tilde{X}; M \tilde{X}_n) = I_A(M^{-1} \tilde{X}; \tilde{X}_n) \]
  i.e. \[ M^T \Gamma M = \Gamma \] for the first \(3 \times 3\) part \( (8) \)
  \[ M^T \bar{U}_0 = \bar{U}_0 \] \( (9) \)

(Yu et al., *Medical Image Analysis*, 10:495-508, 2006)
Understanding of Decorrelation

- Why?
  relative position change of scatterers during convolution

- When?
  - Decorrelation predictable using Eqs. (8) and (9)
  - e.g. lateral rotation or deformation of scatterers will cause decorrelation.

- Decorrelation compensation:
  ill-posed inverse problem as much more unknowns than the data available

- What can we do now?
Related Works I

- Using high frame rate imaging method
  - O’Donnell et al.: 200Hz for 2-D
  - Ultrafast ultrasound imaging: up to 10000 Hz for 2-D
  - Low decorrelation between neighboring frames
  - Large deformation analysis still possible by accumulating many frames

- Limitations
  - high frame rate in 3-D: only 20~28 Hz
  - echocardiography: large deformation between frames
  - decorrelation compensation untouched
Related Works II

- Using additional constraints
  - tissue incompressibility model \((\text{Skovoroda et al. TUFFC 1999})\)
  - deformable mesh method \((\text{Yeung et al. TMI 1998})\)
  - finite element method \((\text{Palmeri et al. TUFFC 2005})\)
  - regularized multi-scale estimation method \((\text{Pellot-Barakat et al. TMI 2004})\)

- Limitations
  - approximate solution
  - decorrelation compensation untouched
Related Works III

- Modeling image variations caused by tissue motion
  - 1-D: temporal stretching

- Common property
  Modeling variations as shift plus scaling.

- Limitations
  Will fail when deformation is large → Why?
Principle of Companding Method

Tissue before deformation:  \( I(\vec{X}; \vec{X}_n) = \sum_{n=1}^{N} T_n(\vec{X}; \vec{X}_n) \ast H(\vec{X}). \)

Tissue after deformation:  \( I(\vec{X}; \vec{Y}_n) \) with \( \vec{Y}_n = M \vec{X}_n + \vec{T}. \)

Tissue before and after deformation

Effect of companding in tracking

\( I(\vec{X}; \vec{X}_n) \)

\( I(\vec{X}; \vec{Y}_n) \)

\( I(M \vec{X} + \vec{T}; \vec{Y}_n) \)
Mathematical Details

\[ T_n(\vec{X}; \vec{Y}_n) = a_n \delta(\vec{X} - (M\vec{X}_n + \vec{T})) \]
\[ = \frac{1}{|M|} a_n \delta(M^{-1}(\vec{X} - \vec{T}) - \vec{X}_n) \]
\[ = \frac{1}{|M|} T_n(M^{-1}(\vec{X} - \vec{T}); \vec{X}_n). \]

\[ T_n(M\vec{X} + \vec{T}; \vec{Y}_n) = \frac{1}{|M|} T_n(\vec{X}; \vec{X}_n). \]

\[ I(M\vec{X} + \vec{T}; \vec{Y}_n) \]
\[ = \sum_{n=1}^{N} \int_{R^3} T_n(M\vec{X} + \vec{T} - \vec{X}'; \vec{Y}_n) H(\vec{X}') d\vec{X}' \]
\[ = \sum_{n=1}^{N} \int_{R^3} T_n \left( M(\vec{X} - M^{-1}\vec{X}') + \vec{T}; \vec{Y}_n \right) H(\vec{X}') d\vec{X}' \]

\[ I(M\vec{X} + \vec{T}; \vec{Y}_n) \ast H(\vec{X}) = I(\vec{X}; \vec{X}_n) \ast H(M\vec{X}). \]

\[ \sum_{n=1}^{N} \int_{R^3} \frac{1}{|M|} T_n \left( \vec{X} - M^{-1}\vec{X}'; \vec{X}_n \right) H(\vec{X}') d\vec{X}' \]

\[ X'' = M^{-1} X' \]
\[ \sum_{n=1}^{N} \int_{R^3} T_n \left( \vec{X} - X''; \vec{X}_n \right) H(M\vec{X}'') d\vec{X}'' \]
\[ = \sum_{n=1}^{N} T_n(\vec{X}; \vec{X}_n) \ast H(M\vec{X}). \]

Valuable insight!

Basis of new method.

Explains the success AND limitation of the companding method.
Algorithm 1  Coupled Filtering Method

Input:
$I(\bar{X}; \bar{X}_n)$: RF image block before tissue deformation;
$I(\bar{X}; \bar{Y}_n)$: RF image block after tissue deformation.

Parameter:
search range and search step of $M$ and $\bar{T}$

Output:
The optimal $M_o$ and $\bar{T}_o$ that best satisfy
$I(M\bar{X} + \bar{T}; \bar{Y}_n) \ast H(\bar{X}) = I(\bar{X}; \bar{X}_n) \ast H(M\bar{X})$.

/* Phase 1-Initialization */
Set $M_o = I$, $\bar{T}_o = 0$, and the threshold $C_m = 0$.
/* We use the correlation coefficient as the metric */

/* Phase 2-Search for Solution */
while $M$ and $\bar{T}$ still in the search range do
  $F_1 \leftarrow I(M\bar{X} + \bar{T}; \bar{Y}_n) \ast H(\bar{X})$.
  $F_2 \leftarrow I(\bar{X}; \bar{X}_n) \ast H(M\bar{X})$.
  $C \leftarrow \frac{\sum_{\bar{X}} (F_1 \cdot F_2)}{\sqrt{\sum_{\bar{X}} F_1^2 \cdot \sum_{\bar{X}} F_2^2}}$.
  if $C > C_m$ then
    $C_m \leftarrow C$, $M_o \leftarrow M$, $\bar{T}_o \leftarrow \bar{T}$
  end if
  Update $M$ and $\bar{T}$ based on the pre-defined search step
end while

return $M_o$ and $\bar{T}_o$
Comparison Using Simulation

- Use the linear convolution model

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Central Frequency of Ultrasound Transducer</td>
<td>4MHz (MegaHertz)</td>
</tr>
<tr>
<td>Ultrasound Speed</td>
<td>1540m/s (Meter/Second)</td>
</tr>
<tr>
<td>Resolution Cell Size</td>
<td>1mm × 1mm × 0.5mm (millimeter)</td>
</tr>
<tr>
<td>Range Along x-Axis</td>
<td>-5mm ~ 5mm</td>
</tr>
<tr>
<td>Range Along y-Axis</td>
<td>-5mm ~ 5mm</td>
</tr>
<tr>
<td>Range Along z-Axis (Beam Direction)</td>
<td>0mm ~ 5mm</td>
</tr>
<tr>
<td>Voxel Size</td>
<td>0.0625mm x 0.0625mm x 0.03125mm</td>
</tr>
</tbody>
</table>

- Deform the scatterers and recalculate the image

\[
M = \begin{bmatrix}
1 + \varepsilon_x \\
1 + \varepsilon_y \\
1 + \varepsilon_z 
\end{bmatrix}
\]

\[\varepsilon_x = \varepsilon_y = -0.5 \varepsilon_z\]
Correlation Coefficient vs. Axial Compression

The graph shows the mean correlation coefficients of correlation coefficients vs. applied strain (%). There are three lines representing different preprocessing methods:

- Black line: no preprocessing
- Blue line: companding
- Red line: coupled filtering

The x-axis represents the applied strain (%) ranging from -10 to 10, while the y-axis represents the mean correlation coefficients ranging from -0.6 to 1.0.
Correlation Coefficient vs. Lateral Rotation

- Mean correlation coefficients
- Rotation angle (degree)

Graph showing correlation coefficients for different preprocessing methods:
- No preprocessing
- Companding
- Coupled filtering
Phantom Data Result

- Phantom Image I
Phantom Data Result

- Phantom Image II
Companding Result

- 5% compression case

mean correlation coefficient: 0.65
Coupled Filtering Result

- 5% compression case

mean correlation coefficient: 0.93
Signal Quality Check
The Computational Challenge: I

**High-dimensional parameter space**!
- $M$: $df = 8$ for 3-D, $df = 3$ for 2-D
- $T$: $df = 3$ for 3-D, $df = 2$ for 2-D

**Speed up of computational at the algorithm level**
- Use multi-scale framework
- Tissue incompressibility constraint

**Speed up of computational at the hardware level**
- **GPU implementation**: 5 hours to analyze a pair of 1001x201 images using the NVIDIA GeForce GTX 580 graphic card.
- **FPGA implementation**: 15 minutes to finish the same analysis using the XILINX Virtex-7 XC7VX690T FPGA card.
The Computational Challenge: II

- Our goal: < 1 minute for 2-D,
  <10 minutes for 3-D

- Apparently, hardware acceleration is not going to be enough.
- We may still have to develop new algorithms.

- Your suggestions are welcome!
Thank you!

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References


- 余维川，梁天柱 “有利于运动估计与特征-运动去相关补偿的方法、装置和系统”。中国专利申请号：ZL 2010 8 0017129.3 授权公告日期：2013.08.21
