AN $L_1$ CONSTRAINED MINIMIZATION APPROACH TO THE HIGH-DIMENSIONAL
MARKOWITZ OPTIMIZATION PROBLEM

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Recently, portfolio selection among a huge number of stocks has become a
vivid research topic. Machine learning and random matrix techniques play very
important roles in this so-called high-dimensional portfolio optimization prob-
lem. We propose an $L_1$ constrained minimization approach to solve the large-
scale Markowitz optimization problem. In particular, we estimate the portfolio
weights $\Sigma^{-1}\mu$ ($\Sigma$ is covariance matrix and $\mu$ is vector of expected return), and
its corresponding expected return $\mu^T\Sigma^{-1}\mu$. Based on these two estimators for
any given risk constraint $\sigma$, we can solve for the sparse portfolio which asymp-
totically yields the maximum expected return and meanwhile satisfies the risk
constraint $\sigma$. We also prove that such portfolio can achieve the sign consis-
tency asymptotically, which means that we select the right stocks(sparsity) in
the right position (sign consistency).