Data-Driven Multiscale Optimization of Energy Systems

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Mathematical Optimization:

\[
\begin{align*}
\min & \quad f(x, y) \\
\text{s.t.} & \quad g(x, y) \leq 0 \\
& \quad h(x, y) = 0 \\
& \quad x \in \mathbb{R}^n, \ y \in \mathbb{Z}^m
\end{align*}
\]

Formulations:
- Nonlinear, nonconvex
- Differential and algebraic equations
- Stochastic
- Mixed Integer
- Multi-objective
Background: Electricity Markets

Multiscale Modeling & Optimization

Dynamic Mixed Integer Optimization

Stochastic Optimization
Paradigm Shift in Energy Technology Landscape

baseload vs. peaking generators

diverse technologies, spectrum of dynamic flexibility

Technologies

Solar
Natural Gas
Energy Storage
Nuclear
Manufacturing
Buildings

Electric Grid

Electric Transmission Lines
California, 2016

Paradigm Shift in Energy Technology Landscape

Key Questions:
How to quantify economic potential for emerging technologies from historical market data?

How to bid into multiscale markets?

How to design more flexible energy systems and more efficient energy markets?
California Energy Price Signals

Annual price distribution for 1-3pm

Day-Ahead Market

Real-Time Market

300% increase during < 1 hour

Energy prices (all market layers) during 4-day horizon

Data from http://oasis.caiso.com
Fourier Analysis of CAISO Energy Prices

\[ \pi(t) = \sum_{k=0}^{N} A_k \sin(\omega_k t) + B_k \cos(\omega_k t) \]

\[ |\pi(j\omega)| = \sqrt{A_k^2 + B_k^2} \]

\( \pi(t) \) = \sum_{k=0}^{N} A_k \sin(\omega_k t) + B_k \cos(\omega_k t)

97% of signal magnitude is from \(10^{-5}\) Hz (day-to-day) and faster frequencies

Dowling and Zavala (2017), *Computers & Chemical Engineering*.
Spatial Price Variations in CAISO

**Integrated Forward Market**
(1-hour intervals)

**Fifteen Minute Market**
(15-min. intervals)

**Real-Time Dispatch Process**
(5-min. intervals)

**Observations:**
- Over **1 trillion** prices for CA system in 2015 (500 GB uncompressed text)
- Faster timescales are most volatile
- Localized volatility at slower timescales
- System-wide volatility at faster timescales

How can a market participant **take advantage of price variations**?
Hierarchical Markets

Ancillary Services
(contingency products)

Electrical Energy
(consumption and generation)

Day-Ahead Market (DAM)
Schedule generators and loads in 1-hour intervals for the next day

Fifteen Minute Market (FMM)
Correct day-ahead schedule due to forecasting errors

Real-Time Dispatch (RTD)
Recenter regulation resources; settle ramping energy

Non-Spinning Reserves
Additional contingency capacity

Spinning Reserves
Fast contingency capacity; supports regulation and RTD

(Power) Regulation
Frequency control by adjusting generators’/loads’ power set-points

Key Finding: Largest revenue opportunities are from fast timescales
### Market Participation Problem

<table>
<thead>
<tr>
<th></th>
<th>Day-Ahead Market</th>
<th>Real-Time Market</th>
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<tbody>
<tr>
<td></td>
<td>Integrated Forward Market (IFM)</td>
<td>Fifteen Minute Market (FMM)</td>
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<tr>
<td></td>
<td>1 hour</td>
<td>15 minutes</td>
</tr>
<tr>
<td><strong>Energy</strong></td>
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<td>✓</td>
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<tr>
<td><strong>Ancillary Services</strong></td>
<td></td>
<td></td>
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<tr>
<td>Regulation Down</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>Regulation Up</td>
<td>✓</td>
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<tr>
<td>Spinning Reserves</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>Non-Spin. Reserves</td>
<td>✓</td>
<td>✓</td>
</tr>
</tbody>
</table>

**Observation:** No previous framework considers **multi-market multi-product** participation.
Optimization Framework for Market Participation

Multiscale Time Discretization

Mathematical abstraction supports:
- Arbitrary technologies
- Arbitrary hierarchical levels & timescales
- Many market designs (e.g., PJM, MISO, CAISO)

Multi-scale market participation involves huge number of decisions
- What products to sell/buy and at what times?
- Constrained by resource’s physics
- **Example:** 1-year horizon → 300,000 variables

Dowling, Kumar, Zavala (2017), *Applied Energy*
Market Participation Model

\[ \mathcal{A} := \{s, n, r^+, r^-\}, \quad \ell \in \mathcal{L} \]

**Time**

\[ \mathcal{T}_\ell := \{1, \ldots, N_\ell\}, \quad \ell \in \mathcal{L} := \{3, 2, 1, 0\} \]

\[ \mathcal{T}^* := \mathcal{T}_{|\mathcal{L}|} \times \cdots \times \mathcal{T}_2 \times \mathcal{T}_1 \times \mathcal{T}_0 \]

\[ t^*_\ell(t) \in \mathcal{T}^*_\ell \]

**Market Products**

- \( A_{t^*_\ell}(t) \): Ancillary service capacity for level \( \ell \in \mathcal{L} \)
- \( \tilde{E}_{t^*_\ell}(t) \): Energy sold in market at level \( \ell \in \mathcal{L} \)
- \( E_{t^*_\ell}(t) \): Energy purchased in market at level \( \ell \in \mathcal{L} \)

**Bounds**

- \( 0 \leq E_{t^*_\ell}(t), \tilde{E}_{t^*_\ell}(t), s_{t^*_\ell}(t), n_{t^*_\ell}(t) \leq 1 \)
- \( 0 \leq r^+_{t^*_\ell}(t) \leq \rho^\text{max}_+ \)
- \( 0 \leq r^-_{t^*_\ell}(t) \leq \rho^\text{max}_- \)

**Net Energy**

\[ E_{t^*_3}(t) = \tilde{E}_{t^*_3}(t) + \sum_{\ell \in \mathcal{L}} \left( \tilde{E}_{t^*_\ell}(t) - E_{t^*_\ell}(t) \right) \]

**Ramping Limit**

\[ -\rho_{\text{elec}} \Delta t^3 \leq E_{t^*_3}(t) - E_{t^*_3(t-1)} \leq \rho_{\text{elec}} \Delta t^3 \]

**Energy Revenue**

\[ R_E = \Lambda^e \sum_{t \in \mathcal{T}^*} \sum_{\ell \in \mathcal{L}} \Delta t_{\ell} \pi_{t^*_\ell}(t) \left( \tilde{E}_{t^*_\ell}(t) - (1 + \epsilon) E_{t^*_\ell}(t) \right) \]

**Ancillary Services Revenue**

\[ R_{\text{AS}} = \Lambda^e \sum_{a \in \mathcal{A}} \sum_{t \in \mathcal{T}^*} \sum_{\ell \in \mathcal{K}} \left( \pi_{a, t^*_\ell}(t) a_{t^*_\ell}(t) \right) \]

Dowling, Kumar, Zavala (2017), *Applied Energy*
Computational Framework: Data + Physical Models + Optimization

Automated Data Acquisition

Resource Physics Model

Model Template
- Express as algebraic equations
- Link to market model via energy variables

Examples
- batteries
- industrial utilities
- solar thermal

Electricity Market Models

Markets
Multiscale Time Discretization

Resource
Model Abstraction
Ramp rate
Max. regulation capacity
Min. operating capacity

Energy Sales
Energy Purchases
Ancillary Service Sales

Numerical Optimization
Easy to Parallelize

Visualization

Dowling, Kumar, Zavala (2017), Applied Energy
Energy Storage Investments in California

Mandate: procure 1,325 MW of storage by 2020

- **Escondido, CA**: San Diego Gas & Electric
  - Opened: Feb. 2017
  - Storage Size: 120 MWh
  - Power Rating: 30 MW
  - Cost: Not Disclosed
  - Supplier: AES
  - Technology: Li-ion

- **El Cajon, CA**: San Diego Gas & Electric
  - Opened: Feb. 2017
  - Storage Size: 30 MWh
  - Power Rating: 7.5 MW
  - Cost: Not Disclosed
  - Supplier: AES
  - Technology: Li-ion

- **Chino, CA**: Southern California Edison
  - Storage Size: 80 MWh
  - Power Rating: 20 MW
  - Cost: $45 million (estimate)
  - Supplier: Tesla
  - Technology: Li-ion
Battery Energy Storage System

CAISO Markets
(IFM, FFM, RTDP)

Decision Variables:
- Market participation schedule

Constraints:
- California market rules
- Battery physics

Input Data:
- Which markets/products to transact?
- Location in CAISO
- Storage size (in hours)

Time horizon: 1 year

Tesla PowerPack System
- 88% to 89% round trip efficiency
- 50 kW to 2.5 MW
- 2 hr to 6 hr of storage
- 900 $/kW to 2,700 $/kW

Goal: Maximize Revenue

Data from [www.tesla.com/powerpack](http://www.tesla.com/powerpack), accessed March 2017
What Size and How to Interact with Markets?

Key Findings:
- Participate in multiple markets
- Transact multiple products
- Smaller energy to power ratios

Median Payback

Payback Period (years) vs. Storage Size (hours)

- **DAM (1-hr prices)**
- **FMM (15-min prices)**
- **RTDP (5-min prices)**

- DAM only, Energy only
- DAM + RTM, Energy + AS
- 100, Energy + AS
- 010, Energy + AS
- 010, Energy
- 011, Energy + AS
- 011, Energy
- 001, Energy
- 111, Energy
- 111, Energy + AS

DAM (1-hr prices)

FMM (15-min prices)

RTDP (5-min prices)
Where to Locate?

Full Market Participation
DAM and RTM
Energy and Ancillary Services

Storage Size: 1 hour

Investment: $570,000 / MW

Revenue: $330,000 to $550,000 / MW / yr

Computational Stats:
- 6,600 nodes analyzed
- 10s to 24s per node (Gurobi 7.0)
- 200 CPU-hours (serial) for map

Based on 2015 Market Prices

Is central CA optimal location for storage from grid operator’s perspective? (e.g., maximize overall reliability, minimize overall system cost)
How important are degradation effects?

Goal: Maximize Net Present Value

Decision Variables:
- Market participation schedule
- Storage size (design)

Constraints:
- California market rules
- Battery physics

Input Parameters:
- Which markets/products to transact?
- Replacement horizon \((N)\)
- Degradation rate \((\epsilon_d)\)

Problem Stats. \((N = 5\) yrs\):
- Linear program
- 3 to 5 million variables
- 4 to 7 million constraints
- 2 CPU-hours (mean) per instance


Cost Data: Dicorato et al (2012), *IEEE Trans. STE*
Degradation Effects for Sodium Sulfur Batteries

Consider 5-year replacement strategy

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</tr>
<tr>
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<td>4,000</td>
</tr>
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<td>High</td>
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</tbody>
</table>

Key Findings
- **AS and RTM** drive economics
- Only **10%** NPV improvement from *technology breakthrough*
- NPV is most sensitive to **market participation mode**
Degradation Effects for Sodium Sulfur Batteries

Consider 5-year replacement strategy

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Observations
- Need larger battery for AS, RTM
- Low degradation (red): largest battery
- High degradation (green): 10% lower NPV, 2x smaller battery
- All cases: less than 1 hour of storage is optimal
When to Replace?

Observation: optimization exploits degrees of freedom in market participation to mitigate degradation

\[
PPY = \frac{NPV}{\sum_{y=1}^{N} (1 - r)^{y-1}}
\]

Observation: optimization exploits degrees of freedom in market participation to mitigate degradation
Concentrated Solar Power Technologies

Power Tower

- Cost: $2.2 billion
- 1,100 GWh / year (100,000 U.S. homes)
- Direct steam generation (1050 °F)
- Land use: 14 km²
- No Storage

Ivanpah Solar, California

Parabolic Trough

- Cost: $600 million
- 210 GWh / year (20,000 U.S. homes)
- Organic heat transfer fluid (400 °F)
- Land use: 2.5 km²
- No Storage

Shams I, United Arab Emirates

Notre Dame Campus: 5.1 km²
Day-Ahead Energy Prices and Solar Irradiance

January – March, 2015

April – June, 2015

July – September, 2015

October – December, 2015

Dowling, Zheng, Zavala (2017), Renewable Sustainable Energy Reviews
Optimal Scheduling and Control of Solar Thermal Systems

Decision variables: Market participation schedule, mass and energy flows
Input data: Market prices, solar irradiance, solar field and thermal storage sizes
Discrete Decisions and Operating Modes

Generating: \( y_{t_1}^e(t) \in \{0, 1\}^{N_1 \times N_0} \)

Spinning: \( y_{t_1}^s(t) \in \{0, 1\}^{N_1 \times N_0} \)

Non-spinning: \( y_{t_1}^n(t) \in \{0, 1\}^{N_1 \times N_0} \)

\[
y_{t_1}^e(t) + y_{t_1}^s(t) + y_{t_1}^n(t) \leq 1
\]

Generating Mode

**Regulation Capacity Limits**

\[
\sum_{\ell \in \mathcal{L}} \left( r_{t_\ell}^+ + r_{t_\ell}^- \right) \leq \rho_{\text{reg}} y_{t_1}^e(t)
\]

\[
\sum_{\ell \in \mathcal{L}} \left( r_{t_\ell}^+ + s_{t_\ell}^* + n_{t_\ell}^* \right) \leq \rho_{\text{reg}}
\]

\[
\theta_r \hat{E}_{t_3}^* + \sum_{\ell \in \mathcal{L}} \tilde{E}_{t_\ell}^* \geq \sum_{\ell \in \mathcal{L}} r_{t_\ell}^-
\]

Maximum Capacity

\[
E_{t_3}^* + \sum_{\ell \in \mathcal{M}} \left( s_{t_\ell}^* + n_{t_\ell}^* + r_{t_\ell}^+ \right) \leq 1
\]

Minimum Capacity

\[
x_{t_3}^* \geq 0, \quad x_{t_3}^* \geq \left( \sum_{\ell \in \mathcal{L}} r_{t_\ell}^- \right) - \theta_r
\]

\[
E_{t_3}^* \geq \lambda y_{t_1}^e(t) + x_{t_3}^*
\]
Optimal Scheduling and Control of Solar Thermal Systems

Problem Size (1-day horizon):
8250 continuous variables
72 binary variables
8748 linear constraints
2916 nonlinear constraints

\[
\max_{z(t), x(t), u(t), y(t)} \int_{0}^{T_f} \pi^T u(t) \, dt \\
\text{s.t. } Au(t) + By(t) \leq 0 \\
dz \over dt = f(z(t), x(t), u(t)) \\
g(z(t), x(t), u(t)) = 0 \\
z \leq z(t) \leq \bar{z} \\
x \leq x(t) \leq \bar{x} \\
u \leq u(t) \leq \bar{u}
\]

Revenue
Market rules\(^1\)
Startup and shutdown limits\(^2\)
CSP dynamics (nonconvex)\(^3,4\)
Bounds

\(^1\)Dowling, Kumar, Zavala (2017), *Applied Energy*
\(^2\)Domínguez, Baringo, Conejo (2012), *Applied Energy*
\(^3\)Patnode, (2006), *UW-Madison*
Multiscale Decomposition Algorithm

**Scheduling Problem**
(mixed integer linear program)

\[
\begin{align*}
\text{max} & \quad \int_0^{T_f} \pi^T u(t) \, dt \\
\text{s.t.} & \quad Au(t) + By(t) \leq 0 \\
& \frac{d\hat{z}}{dt} = \hat{f}(\hat{z}(t), x(t), u(t)) \\
& \hat{g}(\hat{z}(t), x(t), u(t)) = 0 \\
& \hat{z} \leq \hat{z}(t) \leq \tilde{z} \\
& \bar{x} \leq x(t) \leq \bar{x} \\
& \bar{u} \leq u(t) \leq \bar{u}
\end{align*}
\]

\[\hat{z}(t) = [M_1, M_2, E_1, E_2]^T\]

**Dynamic Optimization Problem**
(nonlinear program)

\[
\begin{align*}
\text{max} & \quad \int_0^{T_f} \pi^T u(t) \, dt \\
\text{s.t.} & \quad Au(t) + By(t) \leq 0 \\
& \frac{dz}{dt} = f(z(t), x(t), u(t)) \\
& g(z(t), x(t), u(t)) = 0 \\
& \bar{z} \leq z(t) \leq \tilde{z} \\
& \bar{x} \leq x(t) \leq \bar{x} \\
& \bar{u} \leq u(t) \leq \bar{u} \\
& y(t) \text{ is fixed}
\end{align*}
\]

\[z(t) = [M_1, M_2, T_1, T_2]^T\]

**Operating Schedule**
(1-hr resolution)

\[y(t)\]

**State Trajectories**
(5-min resolution)

\[x(t), z(t)\]

**Linear Surrogate Model**

\[M_{i,t} (\tilde{T}_{i,t}^{avg} - \delta_T) \leq E_{i,t} C_p \leq M_{i,t} (\tilde{T}_{i,t}^{avg} + \delta_T), \quad i \in \{1, 2\}\]
### Computational Performance

<table>
<thead>
<tr>
<th>Market Products</th>
<th>Horizon</th>
<th>Algorithm</th>
<th>Objective</th>
<th>CPU-Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>Energy Only</td>
<td>1 day</td>
<td>(AR1)</td>
<td>$ 621.4</td>
<td>44.3 s</td>
</tr>
<tr>
<td></td>
<td></td>
<td>B-BB</td>
<td>$ 621.4</td>
<td>271.1 s</td>
</tr>
<tr>
<td></td>
<td></td>
<td>B-QG</td>
<td>$ 621.4</td>
<td>111.7 s</td>
</tr>
<tr>
<td></td>
<td></td>
<td>B-Hyb</td>
<td>$ 621.4</td>
<td>71.6 s</td>
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<tr>
<td></td>
<td></td>
<td>B-OA</td>
<td>$ 589.8</td>
<td>15,976 s</td>
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<tr>
<td>Energy Only</td>
<td>2 days</td>
<td>(AR1)</td>
<td>$ 1231.5</td>
<td>131.5 s</td>
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<td></td>
<td></td>
<td>B-BB</td>
<td>$ 1256.5</td>
<td>21,656.9 s</td>
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<tr>
<td></td>
<td></td>
<td>B-QG</td>
<td>$ 1255.4</td>
<td>3611.7 s</td>
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<tr>
<td></td>
<td></td>
<td>B-Hyb</td>
<td>$ 1191.0</td>
<td>3229.3 s</td>
</tr>
<tr>
<td>Energy Only</td>
<td>3 day</td>
<td>(AR1)</td>
<td>$ 1869.1</td>
<td>333.3 s</td>
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<td></td>
<td></td>
<td>B-BB</td>
<td>$ 1783.2</td>
<td>21,760.6 s</td>
</tr>
<tr>
<td></td>
<td></td>
<td>B-QG</td>
<td>$ 1886.3</td>
<td>22,036.7 s</td>
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<tr>
<td></td>
<td></td>
<td>B-Hyb</td>
<td>$ 1896.4</td>
<td>7159.1 s</td>
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<tr>
<td>All Products</td>
<td>1 day</td>
<td>(AR1)</td>
<td>$ 670.9</td>
<td>31.7 s</td>
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<td>B-BB</td>
<td>$ 671.1</td>
<td>989.4 s</td>
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<td>B-QG</td>
<td>$ 671.1</td>
<td>142.4 s</td>
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<td></td>
<td>B-Hyb</td>
<td>$ 671.1</td>
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<tr>
<td>All Products</td>
<td>2 days</td>
<td>(AR1)</td>
<td>$ 1369.8</td>
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<td>B-BB</td>
<td>$ 1366.6</td>
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<td>B-QG</td>
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<td>B-Hyb</td>
<td>$ 1369.2</td>
<td>4,543.8 s</td>
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</table>

**Observations:**

- Decomposition algorithm (AR1) is 3 to 100x faster than off-the-shelf MINLP solvers
- Objective (revenue) converges in 10 or fewer iterations, within 3% or less of best value found

*Dowling, Zheng, Zavala (2018), AIChE Journal*
Optimal Control and Market Participation Strategy

Regulation Down

Regulation Up

Spinning Reserves

Electrical Generation

Breakdown of Market Revenues (extrapolated to 1 year)

Storage: 8 hr, Solar Multiple: 1.5

Physical Models + Data + Optimization to Elucidate Market Incentives

Proposed mathematical abstraction and computational framework
- Supports arbitrary technologies and market structures

Batteries are an attractive investment
- **Payback in 1 to 2 years** with optimized full market participation
- Smaller storage sizes are optimal
- Install in **central CA** (near Fresno)
- Only 10% higher NPV with 10x slower degradation

New opportunities for solar thermal
- Up to 50% higher revenues with ancillary services
- Decomposition algorithm for simultaneous scheduling, control, and market participation
What about uncertainty?

Results generated with **perfect information**. Thus revenue estimates are **upper bounds**. Challenge: How to model **multiscale spatial-temporal uncertainty** in huge datasets?
Brief Agenda

Background: Electricity Markets

Multiscale Modeling & Optimization

Dynamic Mixed Integer Optimization

Stochastic Optimization

Outlook
Optimal PID Controller Tuning under Uncertainty

How to **systematically tune controller gains** to accommodate wide range of operational events?
- Set-point changes
- Exogenous disturbances
- Physical constraints

Cost Function

\[
\begin{align*}
\min_{\pi} & \quad \phi(x_T, y_T, \epsilon_T, u_T, \pi) \\
\text{Dynamics} & \quad \dot{x}(t) = f(x(t), u(t), d(t), p), \quad t \in \mathcal{T} \\
& \quad y(t) = h(x(t), d(t), p), \quad t \in \mathcal{T} \\
& \quad \epsilon(t) = \bar{y}(t) - y(t), \quad t \in \mathcal{T} \\
\text{PID Control Laws} & \quad u(t) = \text{diag}(K_P)\epsilon(t) + \text{diag}(K_I)I(t) + \text{diag}(K_D)\dot{\epsilon}(t), \quad t \in \mathcal{T} \\
& \quad \dot{I}(t) = \epsilon(t), \quad t \in \mathcal{T} \\
\text{Constraints} & \quad 0 \leq g(x(t), u(t), y(t), \pi), \quad t \in \mathcal{T} \\
& \quad x(0) = x_0.
\end{align*}
\]

Gains

\[
\pi = (K_P, K_I, K_D)
\]

(Random) Data

\[
\xi = (x_0, \bar{y}_T, d_T, p)
\]

**Compact Form**

\[
\begin{align*}
\min_{\pi} & \quad \varphi(\pi, \xi) & \quad \text{Cost Function} \\
\text{s.t.} & \quad \pi \in \Pi(\xi) & \quad \text{Feasible Set}
\end{align*}
\]

Renteria, Cao, Dowling, Zavala (2017), *AIChE J.*
Stochastic Programming

a.k.a Optimization Under Uncertainty or Stochastic Optimization

\[
\min_\pi \mathbb{E}_\Xi [\varphi(\pi, \Xi)] \\
\text{s.t. } \pi \in \Pi(\Xi), \; \text{a.s.}
\]

Data are modeled as random variables \(\Xi\) (with associated density \(p_\Xi(\cdot)\))

\(\mathbb{E}_\Xi[\cdot]\) is the expectation operator

Constraint \(\pi \in \Pi(\xi)\) must hold for all possible realizations \(\xi\)

Discrete (Finite-Dimensional) Approximation

\[
\min_\pi \sum_{\xi \in \Omega} w(\xi) \varphi(\pi, \xi) \\
\text{s.t. } \pi \in \Pi(\xi), \; \xi \in \Omega.
\]

\(w(\xi) \in [0, 1]\) is the probability of realization \(\xi \in \Omega\)

\(|\Omega|\) is the number of scenarios

Renteria, Cao, Dowling, Zavala (2017), *AIChE J.*
Sample Average Approximation (SAA) and Statistical Inference

How much data (scenarios) are needed to ensure optimality with a high level of confidence?

Confidence interval for upper bound:

Consider $j = 1, \ldots, T$ data batches, each containing $i = 1, \ldots, \bar{N}$ i.i.d. realizations

For each data batch, compute $\hat{q}_N^j = \frac{1}{N} \sum_{i=1}^{\bar{N}} \varphi(\pi, \xi_{i,j})$ and estimator $\bar{q}_T(\pi) = \frac{1}{T} \sum_{j=1}^{T} \hat{q}_N^j(\pi)$

$$
\mathbb{P} \left( \bar{q}_T(\pi) - z_{\alpha/2} \frac{\bar{s}_T(\pi)}{\sqrt{T}} \leq q(\pi) \leq \bar{q}_T(\pi) + z_{\alpha/2} \frac{\bar{s}_T(\pi)}{\sqrt{T}} \right) = 1 - \alpha
$$

$$
\bar{s}_T(\pi) = \sqrt{\frac{1}{T-1} \sum_{j=1}^{T} (\hat{q}_N^j(\pi) - \bar{q}_T(\pi))^2}
$$

Confidence interval for lower bound:

Consider $k = 1, \ldots, T$ (different) data batches containing $\bar{N}$ i.i.d. realizations

$$
\mathbb{P} \left( q_T - z_{\alpha/2} \frac{\bar{s}_T}{\sqrt{T}} \leq \mathbb{E}[\min_{\pi} \hat{q}_N(\pi)] \leq q_T + z_{\alpha/2} \frac{\bar{s}_T}{\sqrt{T}} \right) = 1 - \alpha
$$

$$
q_T = \frac{1}{T} \sum_{k=1}^{T} \min_{\pi} \hat{q}_N^k(\pi) \quad \bar{s}_T = \sqrt{\frac{1}{T-1} \sum_{k=1}^{T} (\min_{\pi} \hat{q}_N^k(\pi) - q_T)^2}
$$
Sample Average Approximation (SAA) and Statistical Inference

How much data (scenarios) are needed to ensure optimality with a high level of confidence?

Confidence interval for upper bound:

\[
P \left( \bar{q}_T(\pi) - \frac{z_{\alpha/2}}{\sqrt{T}} \frac{\bar{s}_T(\pi)}{\bar{s}_T(\pi)} \leq q(\pi) \leq \bar{q}_T(\pi) + \frac{z_{\alpha/2}}{\sqrt{T}} \frac{\bar{s}_T(\pi)}{\bar{s}_T(\pi)} \right) = 1 - \alpha
\]

Confidence interval for lower bound:

\[
P \left( \frac{q_T - z_{\alpha/2}}{\frac{s_T}{\sqrt{T}}} \leq \mathbb{E}_{\pi} \min_{\pi} \hat{q}_N(\pi) \leq \frac{q_T + z_{\alpha/2}}{\frac{s_T}{\sqrt{T}}} \right) = 1 - \alpha
\]
Risk Metrics and Conditional Value-at-Risk

\[
\min_{\pi, \nu} \mathbb{E}_{\Xi} \left[ \nu + \alpha^{-1}[\varphi(\pi, \Xi) - \nu]_+ \right]
\]

s.t. \( \pi \in \Pi(\Xi), \quad \text{a.s.} \)

CVaR Optimal with \( \alpha = 0.01 \)

Suboptimal Alternative

Renteria, Cao, Dowling, Zavala (2017), AIChE J.
Sparse Grids for Robustness (Worst-Case)

<table>
<thead>
<tr>
<th>$\Omega$</th>
<th>$\max_{\xi \in \Omega} \varphi(\xi)$</th>
<th>$K_P$</th>
<th>$K_I$</th>
<th>$K_D$</th>
<th>$\max_{\xi \in \Omega} \varphi(\xi)$</th>
<th>$K_P$</th>
<th>$K_I$</th>
<th>$K_D$</th>
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<td>1.66</td>
<td>6.80</td>
</tr>
</tbody>
</table>

Renteria et al, AIChE J.
Scalability and High-Performance Computing

Standard Form:
\[
\min_{\pi \in \Pi, y_\xi \in Y_\xi} \sum_{\xi \in \Omega} f_\xi(\pi, y_\xi) \\
s.t. \quad c_\xi(\pi, y_\xi) = 0, \; \xi \in \Omega
\]

Lifted Form:
\[
\min_{\{\pi_\xi\} \in \Pi \times Y_\xi} \sum_{\xi \in \Omega} f_\xi(\pi_\xi, y_\xi) \\
s.t. \quad c_\xi(\pi_\xi, y_\xi) = 0, \; \xi \in \Omega \\
\pi_{\xi+1} = \pi_\xi, \; \xi \in \Omega \setminus \{|\Omega|\}
\]

Block Bordered Diagonal Form:
\[
\begin{bmatrix}
K_\pi & B_1^T & B_2^T & \cdots & B_{|\Omega|}^T \\
B_1 & K_1 & & & \\
B_2 & & K_2 & & \\
\vdots & & \ddots & \ddots & \\
B_{|\Omega|} & & & & K_{|\Omega|}
\end{bmatrix}
\begin{bmatrix}
\Delta w \\
\Delta w_1 \\
\Delta w_2 \\
\vdots \\
\Delta w_{|\Omega|}
\end{bmatrix}
= 
\begin{bmatrix}
r_\pi \\
r_1 \\
r_2 \\
\vdots \\
r_{|\Omega|}
\end{bmatrix}
\]

Results with PIPS-NLP, which uses a Schur-complement decomposition.

| | \Omega| = 1000 scenarios |
|---|---|
| # Cores | Time [sec] |
| 0 | 4000 |
| 5 | 3000 |
| 10 | 2000 |
| 15 | 1000 |
| 20 | 5000 |
| 25 | 0 |

Renteria, Cao, Dowling, Zavala (2017), AIChE J.
Interface of Uncertainty Quantification and Stochastic Programming

Stochastic Programming
+ Explicitly incorporate uncertainty into decision-making
+ CVaR captures risk aversion (extreme tail)
+ Leverage model structure (including sparsity, derivatives)
+ Scalable algorithms exploit problem structure
+ Approximations: Sparse grids, SAA
- Requires well defined probability space

Uncertainty Quantification
+ Provides frameworks to determine probability space
  • Random processes (e.g., weather for renewables)
  • Uncertainty from model simplifications (e.g., Kennedy & O’Hagan)
Mathematical Optimization:

\[
\min \quad f(x, y) \\
\text{s.t.} \quad g(x, y) \leq 0 \\
\quad h(x, y) = 0 \\
\quad x \in \mathbb{R}^n, \; y \in \mathbb{Z}^m
\]

Formulations:
- Nonlinear, nonconvex
- Differential and algebraic equations
- Stochastic
- Mixed Integer
- Multi-objective